A New Efficiency Criterion: The Mean-Separated Target Deviations Risk Model

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This paper develops a new risk efficiency model: Mean-Separated Target Deviations (MSTD). Conventional measures of risk do not distinguish between below-target and above-target outcomes, or else impose risk neutrality for above-target outcomes. The model is motivated by the intuition that although investors are comfortable with expected value as a measure of return, they respond in different ways to potential outcomes below a target return than to potential outcomes above a target return. The measure of risk is a weighted sum of below-target deviations and above-target deviations, and reflects skewness in the return distribution. The investor’s risk attitude determines the weights. MSTD is a special case of a von Neumann-Morgenstern expected utility function. With restrictions on parameters, it is a special case of stochastic dominance. Unlike the mean-variance criterion, the MSTD model considers skewness in ranking alternatives. The model is then used to select among alternative means of hedging hard red winter wheat. The additional restrictions of MSTD are successful in producing a smaller efficient set than other criteria.

I. Introduction

Risk and return models are commonly used to analyze choices under uncertainty. The most common risk-return model is the mean-variance (E-V) model, in which return is measured as the mean, and risk as the variance of the outcome distribution. In spite of its computational and graphical advantages (Hazell and Norton 1986, p. 80) and its attractive dichotomy between risk and return (Holthausen 1981), E-V analysis has several well-known theoretical shortcomings.1 Tobin (1957) argues that E-V analysis is relevant when the utility function is quadratic, or when

1 See Fishburn (1977) and Meyer (1987) for a list of relevant articles.
net returns are normally distributed.\textsuperscript{2} Two main limitations of quadratic utility are: 1) the utility function is not monotonically nondecreasing, and 2) it displays increasing absolute risk aversion. Further, the assumption of normality often does not hold as actual returns are often skewed and leptokurtic.

Recognizing problems with the E-V criterion, alternative efficiency criteria have been introduced. These criteria include E-S (Expected Value-Semivariance) (Markowitz 1952; Mao 1970; Porter 1974), Target Risk-Return (Fishburn 1977; Holthausen 1981), Mean-Gini criterion (Yitzhaki 1982), First-degree Stochastic Dominance (Quirk and Saposnik 1962), Second-degree Stochastic Dominance (Hanoch and Levy 1969; Hadar and Russell 1969), and Stochastic Dominance with respect to a Function (SDWRF) (Meyer 1977; King and Robison 1983). Although these criteria overcome some of the disadvantages of the E-V criterion, they have some limitations. For example, most require the assumption of everywhere risk aversion.\textsuperscript{3}

Moreover, Fishburn (1977, p. 177) contends that the E-V model uses an unrealistic measure of risk. He noted, following Markowitz (1959), Mao (1970), and others, that investors “...frequently associate risk with failure to attain a target return,” suggesting that a measure of dispersion around a parameter which changes from distribution to distribution—such as variance—is not a suitable measure of risk. To address these shortcomings of the E-V model, Fishburn proposed a mean-risk model which generalized the mean-target semivariance model (Markowitz 1959; Mao 1970; Hogan and Warren 1972; Porter 1974). Fishburn's model measured return as the mean of the outcomes, but defined risk as weighted deviations of outcomes below a target, where the weight was related to the investor's risk preferences. Holthausen (1981) adapted Fishburn's model by using the same measure of risk but defining return as weighted deviations above the target rather than as the mean.

Kraus and Litzenberger (1976) proposed a three moment capital asset pricing model (3-moment CAPM). The 3-moment CAPM is not an efficiency criterion, but rather a model of market equilibrium. The 3-moment CAPM is not applicable to discrete choices. One advantage of the 3-moment CAPM is that it is a rigorous theoretical framework which can be tested empirically. Empirical support, however, for the 3-moment CAPM is mixed (Friend and Westerfield 1980; Barone-Adesi 1985; Lim 1989), but does show that skewness is an important determinant of security returns. An efficiency criterion based on similar assumptions to the 3-moment CAPM would require an asset to have higher mean, lower variance, and higher skewness in order to dominate another asset. Such a criterion is strongly defensible theoretically, but leads to a large efficient set. The criterion proposed here yields a much smaller efficient set, but requires much stronger assumptions about investors' preferences.

Our paper builds on Fishburn's and Holthausen's models rather than Kraus and Litzenberger's by proposing a mean-risk model which generalizes Fishburn's model.

\textsuperscript{2} Meyer (1987) shows that a location-scale condition, of which normality is a special case, is the condition for ensuring that E-V analysis is consistent with expected utility.

\textsuperscript{3} SDWRF can allow risk preferring utility; FSD does not require assumptions about the decision maker's risk attitude.
but keeps the mean as the measure of return. The model developed here is called Mean-Separated Target Deviations Risk Model (MSTD). Like Fishburn's and Holthausen's models, MSTD is based on a specific functional form for utility. Fishburn's model assumes risk neutrality above the target. This restriction is avoided in the MSTD model as it is in Holthausen's model. Holthausen's model measures return as above-target deviations, while MSTD measures return as expected return as in Fishburn's model. Expected return is the most common measure of return and investors are satisfied with it as a measure of return (Baumol 1963). The model is then used to select among several alternative strategies of hedging hard red winter wheat.

The model measures return as expected value, and risk as deviations below a target return minus deviations above the target return, with both kinds of deviation weighted by the investor's risk preferences. As Hanoch and Levy (1969, p. 344) note, "The identification of riskiness with variance, or with any other single measure of dispersion, is clearly unsound. There are many obvious cases where more dispersion is desirable, if it is accompanied by an upward shift in the location of the distribution, or by an increasing positive asymmetry." Alternative risk measures such as mean-target semivariance (Mao 1970; Porter 1974) or weighted below-target deviations (Fishburn 1977; Holthausen 1981) consider information only on outcomes below a target and ignore information on outcomes above the target. MSTD, however, considers information on outcomes both below and above the target. Thus, the proposed measure is affected by the skewness of outcome distributions. Investors should dislike negative skewness and like positive skewness.4

Our paper also goes beyond Fishburn and Holthausen by showing how MSTD can be used with interval analysis similar to SDWRF. Like stochastic dominance with respect to a function (SDWRF), MSTD can effectively provide the efficient set by using appropriate ranges of investor's risk attitude. Unlike SDWRF, however, MSTD allows the risk attitude above the target return to differ from the risk attitude below the target return.

Porter (1974), Fishburn (1977), Holthausen (1981), and Yitzhaki (1982) have shown that their risk efficiency models are congruent with expected utility theory and consistent with stochastic dominance rules. Our paper extends their results to show that the MSTD model is also congruent with expected utility theory and consistent with stochastic dominance rules under some conditions.

The MSTD is then used to select among alternative storage strategies for hard red winter wheat. The choices are discrete so it provides a clear example of where the MSTD method might be useful. Further, the problem is a real one and so it is of interest in itself.

II. The Mean-Separated Target Deviations (MSTD) Risk Model

There is usually a point on the abscissa at which something unusual happens to an individual's utility function (Fishburn and Kochenberger 1979). Therefore, using a target in specifying a utility function seems to be appropriate. Fishburn generalized the Expected Value-Semivariance model by associating risk with below-target

4 Arditti provides evidence concerning investor's preferences for positive skewness.
returns. In Fishburn’s (1977) model, return is measured by expected return and risk is measured by the dispersion below a target,

$$\rho(F) = \int_{-\infty}^{t} \phi(t - \pi) \, dF(\pi),$$  \hspace{1cm} (1)

where $\phi(y)$, for $y \geq 0$, is a nonnegative nondecreasing function of $y$ with $\phi(0) = 0$, and $F(\pi)$ is the probability of that return not exceeding $\pi$. $F(\pi)$ is assumed to be bounded with $F(\pi_1) = 0$ and $F(\pi_2) = 1$ for some real $\pi_1$ and $\pi_2$. A special form of equation (1) is the $\alpha$-$t$ model, in which risk $(r(F))$ is measured by:

$$r(F) = \int_{-\infty}^{t} (t - \pi)^{\alpha} \, dF(\theta), \quad \alpha > 0.$$  \hspace{1cm} (2)

Fishburn showed that the $\alpha$-$t$ model is congruent with the expected utility model under the utility function:

$$U(\pi) = \begin{cases} 
\pi & \text{for all } \pi \geq t \\
\pi - k(t - \pi)^{\alpha} & \text{for all } \pi \leq t \text{ and } k > 0.
\end{cases}$$  \hspace{1cm} (3)

If $\alpha > 1$, the individual is risk averse below $t$, if $\alpha < 1$, the individual is risk seeking below $t$, and if $\alpha = 1$, the individual is risk neutral below $t$.

Holthausen (1981) derived an $\alpha$-$\beta$-$t$ model with both risk and return measured as deviations from a target return so that the utility function for the above-target outcomes need not be linear. Risk in the $\alpha$-$\beta$-$t$ model $(r(F))$ is defined as in Fishburn’s model, but return $(\pi(F))$ is defined as above-target deviations,

$$\pi(F) = \int_{t}^{\infty} \theta(\pi - t) \, dF(\theta),$$  \hspace{1cm} (4)

where $\theta(y)$, for $y \geq 0$, is a nonnegative nondecreasing function in $y$ with $\theta(0) = 0$. A specific form of equation (4) along with risk measure equation (2) gives the $\alpha$-$\beta$-$t$ model in which the return $(R(F))$ is measured as:

$$R(F) = \int_{t}^{\infty} (\pi - t)^{\beta} \, dF(\pi), \quad \beta \geq 0.$$  \hspace{1cm} (5)

Using these measures of risk and return, Holthausen also showed that the $\alpha$-$\beta$-$t$ model is congruent with the expected utility model in which the utility function is:

$$U(\pi) = \begin{cases} 
(\pi - t)^{\beta} & \text{for all } \pi \geq t \\
-k(t - \pi)^{\alpha} & \text{for all } \pi \leq t \text{ and } k > 0.
\end{cases}$$  \hspace{1cm} (6)

where $k$ is a constant for a given utility function, and $\alpha$ and $\beta$ reflect the risk preference of $s$. If $\alpha < 1$ ($\alpha > 1$), then the individual is risk seeking (averse) below target, and if $\beta < 1$ ($\beta > 1$), then the individual is risk averse (seeking) above the target.
Fishburn’s utility function is linear in outcomes above the target, which imposes risk neutrality on above-target returns. Holthausen’s utility function, however, is nonlinear in outcomes above the target. Holthausen’s specification allows the investor to have different risk preferences for outcomes above and below the target. Following Fishburn, the proposed MSTD model measures return as expected value. Holthausen did not include expected value in his measure of return and suggested that its use is redundant. However, Baumol (1963) has noted that investors are generally satisfied with expected value as a measure of return.

Model Specification: The Mean-Separated Target Deviations Model

The Mean-Separated Target Deviations (MSTD) model is motivated by the intuition that although investors are comfortable with expected value as a measure of return, they respond in different ways to potential outcomes below a target return than to potential outcomes above a target return. MSTD separates dispersion into two parts: below-target deviations (BTD) and above-target deviations (ATD). BTD reduce the investor’s expected utility, but ATD increase the investor’s expected utility. Therefore, a higher level of dispersion of a distribution (e.g., variance) does not necessarily lower an investor’s utility.

The general measure of risk ($\Omega(F)$) is:

$$\Omega(F) = \delta \int_{-\infty}^{t} \phi(t - \pi) dF(\theta) - \lambda \int_{t}^{\infty} \Theta(\pi - t) dF(\pi),$$  

where $\delta$ and $\lambda$ are positive constants; $\phi(\theta)$ and $\theta(\pi)$ are nonnegative nondecreasing functions of $\pi$ with $\phi(0) = \theta(0) = 0$, and $F(\pi)$ is the cumulative probability distribution function over outcomes $\pi$.

In many ways, $\Omega$ is a more intuitive definition of risk than measures such as variance in E-V, semivariance in E-S, or below-target returns in Fishburn’s and Holthausen’s models. Fishburn (1977) and Holthausen (1981) used only below-target deviation (the first term in equation (8)) as the measure of risk, and Holthausen used above-target deviation (the second term in equation (8)) as the measure of return. The measure of risk used here, $\Omega$, is below-target deviations (BTD) less above-target deviations (ATD), with both weighted by probability and investor’s risk attitude. $\Omega$ increases as BTD increase and decreases as ATD increase. This implies that the more negatively skewed the distribution, the higher the risk; and the more positively skewed, the lower the risk. Because deviations are measured from the investor’s target return, skewness from the investor’s point of view may be more appropriately measured as skewness around the target. Thus, this risk measure captures the skewness of the probability distribution and, as shown below, is flexible, allowing one to incorporate various levels of risk attitude into the model.

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5 Skewness around the mean is measured as $(1/n) \sum (x - \mu)/\sigma^3$, where $\mu$ is mean and $\sigma$ is standard deviation; and skewness around the target is measured as $(1/n) \sum (x - t)/s^3$, where $t$ is the specified target return and $s$ is standard deviation around the target instead of around the mean.
A specific form of equation (7) is the MSTD model which allows easy estimation. In the MSTD model, risk \((SD_t(F))\) is defined by:

\[
SD_t(F) = \delta \int_{-\infty}^{t} (t - \pi)^{\alpha} dF(\pi) - \lambda \int_{t}^{\infty} (\pi - t)^{\beta} dF(\pi), \quad \alpha, \beta > 0.
\]  

(8)

**Congruence with Expected Utility.** Combining \(\Omega(F)\) from equation (7) with expected value \(E_F\), gives a preference relationship in which the investor's preferences depend only on \(E_F\) and \(\Omega(F)\). This section extends Fishburn's and Holthausen's results to show the relationship between the MSTD model and the expected utility model. Let \(U(E_F, \Omega(F))\) be a real-valued function such that, for all relevant distributions \(F\) and \(G\), \(F\) is preferred to \(G\) if and only if \(U(E_F, \Omega(F)) > U(E_G, \Omega(G))\), where \(U\) is increasing in \(E\) and decreasing in \(\Omega\).

**Theorem 1.** Suppose that, for all bounded distribution functions \(F\) and \(G\), the MSTD model with risk defined by equation (7) is congruent with expected utility in the sense that

\[
\int_{-\infty}^{\infty} u(\pi) dF(\pi) > \int_{-\infty}^{\infty} u(\pi) dG(\pi).
\]

Then with \(u(t) = 0\), \(u(t - 1) = t - 1 - \delta\) and \(u(t + 1) = t + 1 + \lambda\), there exist positive constants \(\delta\) and \(\lambda\) such that:

\[
u(\theta) = \begin{cases} 
\pi - \delta \phi(t - \pi) & \text{for all } \pi \leq t, \\
\pi + \lambda \theta (\pi - t) & \text{for all } \pi \geq t.
\end{cases}
\]

(10)

*(Proof is given in the Appendix)*

Fishburn's utility function in equation (3) is a special case of equation (10) where \(\lambda = 0\).

The expected utility of equation (10) is:

\[
\int_{-\infty}^{\infty} u(\theta) dF(\pi) = E_F - \Omega(F).
\]

(11)

When the MSTD model is used, equation (10) gives:

\[
u(\pi) = \begin{cases} 
\pi - \delta (t - \pi)^{\alpha} & \text{for all } \pi \leq t, \\
\pi + \lambda (\pi - t)^{\beta} & \text{for all } \pi \geq t.
\end{cases}
\]

(12)

\(\delta\) is a unique solution to \(u(t - 1) = t - 1 - \delta\), and \(\lambda\) is a unique solution to \(u(t + 1) = t + 1 + \lambda\). The utility function in equation (12) can display various shapes depending on the values of \(\alpha, \beta, \delta,\) and \(\lambda\). Some possible shapes of the utility function defined in equation (12) are given in Figure 1. The curve with \(\alpha < 1\) is convex or risk preferring, the curve with \(\alpha > 1\) is concave or risk averse,
and the curve with $a = 1$ is linear or risk neutral, below target. The curve with $\beta < 1$ is concave or risk averse, the curve with $\beta > 1$ is convex or risk preferring, and the curve with $\beta = 1$ is linear or risk neutral, above target. Even if $a = 1$ and $\beta = 1$, the individual is still risk averse if $\delta > \lambda$, and risk preferring if $\delta < \lambda$, because the utility function is kinked around the target.

Friedman and Savage (1948) suggested a three-segment utility function which is initially risk averse, then risk preferring, and then risk averse. Kahneman and Tversky (1979) suggested a function which is usually convex below the target and concave above the target. Fishburn and Kochenberger (1979) examined thirty empirically-assessed utility functions with target points. They showed that the mean squared error of power functions as in equation (12) were smaller than that of either the exponential or linear function. They also found that the majority of below-target functions were risk preferring and the majority of above-target functions were risk averse. Therefore, a utility function like equation (12) is a sound candidate for an investor’s utility function.

Consistency of MSTD with Stochastic Dominance Efficiency. Porter (1974) has demonstrated that the E-S efficient set is a subset of second-degree stochastic dominance. Yitzhaki (1982) has shown that the M-G criterion is a necessary condition for first- and second-degree stochastic dominance. Fishburn (1977) and

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6 About two-thirds of the below-target functions were risk preferring and slightly less than three-fifths of the above target functions were risk averse.
Holthausen (1981) have also shown that their models are special cases of the stochastic dominance rules. This section extends Fishburn’s and Holthausen’s results to the MSTD model, and shows that MSTD is also a special case of stochastic dominance rules. The first-, second-, and third-degree stochastic dominance rules are defined as:

\[ F \text{ FSD } G \] if and only if \( F \neq G \) and \( F(\pi) \leq G(\pi) \) for all \( \pi \);

\[ F \text{ SSD } G \] if and only if \( F \neq G \) and \( F_1(\pi) \leq G_1(\pi) \) for all \( \pi \), and

\[ F \text{ TSD } G \] if and only if \( F \neq G \) and \( F_2(\pi) \leq G_2(\pi) \) for all \( \pi \),

letting \( F \text{ FSD } G \), \( F \text{ SSD } G \), and \( F \text{ TSD } G \) denote \( F \) dominates \( G \) by FSD, SSD, and TSD, respectively. Here, \( F_1(\pi) = \int_{-\infty}^{\pi} F(x) \, dx \), so \( F_1(\pi) \) is the area under \( F(\pi) \) up to \( \pi \), and \( F_2 = 2 \int_{-\infty}^{\pi} F_1(x) \, dx \), so \( F_2(\pi) \) is twice the area under \( F_1 \) up to \( \pi \).

The expected utility under distribution \( F \) can be defined as

\[ E(u,F) = \int_{-\infty}^{\infty} u(\pi) \, dF(\pi). \]

By Fishburn’s (1977) Lemma 1, if \( F \text{ FSD } G \), then \( E_F \geq E_G \) and \( E(u,F) \geq E(u,G) \) for every utility function with \( u' \geq 0 \); if \( F \text{ SSD } G \), then \( E_F \geq E_G \) and \( E(u,F) \geq E(u,G) \) for every utility function with \( u' \geq 0 \) and \( u'' \leq 0 \); and if \( F \text{ TSD } G \), then \( E_F \geq E_G \) and \( E(u,F) \geq E(u,G) \) for every utility function with \( u' \geq 0, u'' \leq 0 \), and \( u''' \geq 0 \), where \( u', u'' \), and \( u''' \) are the first, second, and third derivatives of the utility function, respectively. Thus FSD corresponds to nondecreasing utility functions, SSD to nondecreasing and concave utility functions, and TSD to nondecreasing and concave utility functions with \( u''' \geq 0 \). The following theorem shows the relationship between the MSTD model and stochastic dominance rules.

**Theorem 2.** Except for risky alternatives with identical mean and separated target deviations, every MSTD efficient set is a subset of the FSD efficient set for \( \alpha \geq 0 \) and \( \beta \geq 0 \); every MSTD efficient set is a subset of the SSD efficient set for \( \alpha \geq 1 \) and \( 0 \leq \beta \leq 1 \); and every MSTD efficient set is a subset of the TSD efficient set for \( \alpha \geq 2 \) and \( 0 \leq \beta \leq 1 \). (Proof is given in the Appendix.)

### Obtaining Efficient Sets

The Arrow-Pratt absolute risk aversion (ARA) coefficient is

\[
 r(\pi) = -u''(\pi)/u'(\pi),
\]

where \( u' \) and \( u'' \) are the first and second derivatives of a von Neumann-Morgenstern utility function. The ARA coefficient can be used as a measure of an investor’s risk aversion. The larger the ARA coefficient, the more risk averse is the preference. If the ARA coefficient is negative, the investor is risk preferring, and if it is positive, the investor is risk averse. The absolute risk aversion function is invariant to positive linear transformations of the utility function. Therefore, upper and lower bounds on an investor’s ARA function define an interval representation (King and Robison 1981). Like stochastic dominance with respect to a function, MSTD can order risky choices based on an interval representation of the ARA coefficient.

From equation (12), the ARA coefficients below and above \( t \) are,

\[
 r_1 = \alpha(\alpha - 1)\delta(t - \pi)^{\alpha-2} \left[ 1 + \alpha\delta(t - \pi)^{\alpha-1} \right], \quad \pi \leq t,
\]

\[
 r_2 = -\beta(\beta - 1)\lambda(\pi - t)^{\beta-2} \left[ 1 + \beta\lambda(\pi - t)^{\beta-1} \right], \quad \pi \geq t.
\]
Efficiency Criterion

If \( \alpha > 1 \) and \( \beta > 1 \), then the individual is risk averse below \( t(r_1 > 0) \) and risk preferring above \( t(r_2 < 0) \). If \( 0 < \alpha < 1 \) and \( 0 < \beta < 1 \), then the individual is risk preferring below target \( (r_1 < 0) \) and risk averse above target \( (r_2 > 0) \).

The parameters \( \alpha \) and \( \beta \) can be expressed as functions of ARA coefficients below and above the target, respectively. In order to determine the values of \( \alpha \) and \( \beta \), we need to know the target \( t \) and the ARA coefficients at least one point below the target and at one point above the target. Here, the ARA coefficients are assumed to be specific to some corresponding values of \( \tau \). Let \( r_1(\tau_1) \) be an ARA coefficient at a specific level of \( \tau \) below the target and \( r_2(\tau_2) \) be an ARA coefficient at a specific level of \( \tau \) above the target. For simplicity, the levels of \( \tau \), \( t \), and corresponding ARA coefficients can be rescaled so that the difference between the rescaled \( \tau \) and \( t \) becomes unity.\(^7\) For \( \delta = \lambda = 1,\(^8\)

\[
\alpha = \left[ 1 + r_1^*(\tau_1) + \left( (1 + r_1^*(\tau_1))^2 + 4r_1^*(\tau_1) \right)^{1/2} \right] / 2, \quad \tau_1 \leq t, \quad (14)
\]

\[
\beta = \left[ 1 - r_2^*(\tau_2) + \left( (1 - r_2^*(\tau_2))^2 - 4r_2^*(\tau_2) \right)^{1/2} \right] / 2, \quad \tau_2 \geq t,
\]

where \( r_1^*(\tau_1) \) and \( r_2^*(\tau_2) \) are rescaled ARA coefficients at \( \tau_1 \) and \( \tau_2 \), respectively. The more risk averse below target, the bigger the value of \( \alpha \). The more risk preferring above target, the bigger the value of \( \beta \). Therefore, a more negatively skewed return distribution has higher risk with the MSTD model.

In the MSTD model, the distribution \( F \) is preferred over another distribution \( G \), if and only if the expected utility of \( F \) in equation (11) is greater than the expected utility of \( G \). For each combination of \( r_1(\tau_1) \) and \( r_2(\tau_2) \), one distribution is selected.

Also, for given ranges of \( r_1(\tau_1) \) and \( r_2(\tau_2) \), the utility function in equation (12) can take a range, and thus an interval analysis similar to SDWRF is possible. That is, MSTD provides an efficient set for a utility function which lies within specified ranges. The limitation of our approach is that the range of ARA coefficients is specific to one point on the utility function rather than for the entire utility function as with SDWRF. The ranges of \( r_1(\tau_1) \) and \( r_2(\tau_2) \) corresponding to risk averse, risk neutral, and risk preferring investor can be obtained by eliciting the typical investor’s utility functions and computing the absolute risk aversion coefficients. Another alternative is to use ARA coefficients elicited in other studies. Kallberg and Ziembba (1979) and Raskin and Cochran (1986) discuss how to transform risk aversion coefficients from other studies in order to adjust for

\(^7\) Raskin and Cochran (1986) show how to rescale the ARA coefficients with changes in the units of measure. In this analysis, \( \alpha \) and \( \beta \) are determined by the ARA coefficients below and above target, respectively, which are specific to some value of \( \tau \). Therefore, typical levels of gain and loss should be known. Let the typical loss be \( \tau_1 = t - \nu \) and the typical gain be \( \tau_2 = t + \nu \) so that \( |\tau - t| = \nu \). \( \nu \) is a conversion factor. We can standardize the difference \( |\tau - t| \) by dividing \( t, \tau_1, \) and \( \tau_2 \) by the conversion factor. Using this conversion factor, ARA coefficients should be rescaled as \( r_1^*(\tau_1) = \nu r_1(\tau_1) \) and \( r_2^*(\tau_2) = \nu r_2(\tau_2) \).

\(^8\) \( \delta \) and \( \beta \) can also be obtained numerically, that is, the values of \( \alpha \) and \( \beta \) which minimize \([r_1(\tau_1)^\delta - \alpha(\alpha - 1)\delta/(1 + \delta \alpha)]^2\) and \([r_2(\tau_2)^\beta + \beta(\beta - 1)\lambda/(1 + \lambda \beta)]^2\), respectively, are the parameters which reflect at DM’s risk attitude. In this way, \( \delta = \lambda = 1 \) is not required.
changes in the units of measurement. The MSTD efficient set can be obtained using a computer spread sheet. MSTD efficient sets of each interval of ARA coefficients are obtained with a grid search.\(^9\) For given ranges of \(r_1(\pi_1)\) and \(r_2(\pi_2)\), all distributions selected at any point in the grid search are included in the MSTD efficient set.

III. An Empirical Application

This section of our paper selects risk efficient marketing strategies for hard red winter wheat producers. Several routine marketing strategies, including cash-only strategies and routine hedges, are compared with selective strategies using basis predictors. Because hedging shifts price risk in cash and futures markets to basis risk, unexpected changes in basis affect hedging outcomes. This empirical application considers whether producers can increase expected utility by using basis predictors, or indicators, to select marketing strategies.

Basis Indicator

As futures prices and spot prices generally move in the same direction, price risk can be reduced by taking opposite positions in cash and futures markets. However, as Working (1953) noted, the movements of spot and futures prices do not show complete parallelism. A hedger can use this inequality between the movements of spot and futures prices to increase profits by using basis as an indicator for efficient marketing strategies. The unequal movement of spot and futures prices may provide indicators useful for reducing risk and improving profit. Net returns for several routine marketing strategies have been computed each year for 1975–1990, as well as returns developed from selective hedging based on basis indicators. The returns from each of the strategies were compared using E-V, FSD, SSD, SDWRF, E-S, Mean-Gini, and MSTD criteria.

The model used assumes that cash wheat and wheat futures contracts are traded in two time periods: at harvest time (period 0) and when the futures contracts are liquidated (period 1). For the two-period model, net return per bushel is:

\[
F = xC^0 + (1 - x)C^1 + (F^1 - F^0)y - (1 - x)CC, \tag{15}
\]

where \(R = \text{net return (cents/bushel)};\)
\(C^n = \text{cash price in period } n, n = 0, 1;\)
\(F^n = \text{futures contract price in period } n, n = 0, 1;\)
\(x = 1 \text{ if sell wheat at harvest;}\)
\(x = 0 \text{ if store wheat at harvest;}\)

\[^9\] For example, for an individual who is risk averse below target and risk preferring above target, suppose the appropriate intervals of ARA coefficients are (0.56, 2.80) below target and (−1.68, −0.56) above target. Using 0.01 as the grid size, the ARA coefficient below target can be 0.56, 0.57, ..., 2.79, 2.80, and the ARA coefficient above target can be −1.68, −1.67, ..., −0.55, −0.56. The union of all sets obtained using all possible combinations of ARA coefficients below and above targets is the MSTD efficient set for an individual who is risk averse below target and risk preferring above target. Evaluating only at point of ARA coefficients near boundaries of intervals may reduce computing time substantially.
Efficiency Criterion

\[ y = 1 \text{ if buy futures contract at harvest}; \]
\[ y = -1 \text{ if sell futures contract at harvest}; \]
\[ y = 0 \text{ if no action is taken in the futures market}, \]

\[ CC = \text{carrying cost; number of months multiplied by the sum of monthly storage and interest cost}. \]

Three alternative marketing methods are considered.

1. The \textit{CASH} method involves selling wheat at harvest and taking no additional action \((x = 1, y = 0)\). The net return for this marketing method is:

\[ R_i = C_i^0, \quad i = 74, 75, \ldots, 91, \]

where \(R_i\) is net return for year \(i\), and \(C_i^0\) is the local cash price in period 0 (harvest time) in year \(i\).

2. The \textit{SPECULATION} method involves selling wheat and buying futures contracts for speculation at period 0 (at harvest), and then liquidating the futures contract at period 1 \((x = 1, y = 1)\). The net return is:

\[ R_i = C_i^0 + (F_i^1 - F_i^0), \quad i = 74, 75, \ldots, 91, \]

where \(F_i^n\) = price of futures contract in period \(n\) in year \(i\).

3. The \textit{SHORT HEDGING} method involves storing wheat and selling futures contracts in period 0, and then selling wheat and buying back the futures contracts in period 1 \((x = 0, y = -1)\). The net return is:

\[ R_i = C_i^1 - (F_i^1 - F_i^0) - CC, \quad i = 74, 75, \ldots, 91, \]

where \(C_i^1\) is the local cash price in period 1 in year \(i\). A fourth alternative, storing wheat at harvest for later sale \((STORAGE)\), could be considered. However, this will yield approximately the same result as the \textit{SPECULATION} strategy, except that the producer has to pay additional storage and interest charges.

A routine strategy is defined as one which follows a particular marketing method every year. Instead of using the same marketing method every year, basis indicators have been used to select one marketing method from among the three alternatives every year (See Table 1).

Current basis \((CB)\) is defined as the difference between the harvest-time cash price and the harvest-time futures price for a given contract month. Expected basis \((EB)\) is defined as the producer’s expectation of the difference between the cash price and the futures price on the day the producer would liquidate any futures contracts and sell any cash commodity. In period 0, at harvest time, the producer forms an expectation of the period 1 basis.

Various forecasting models have been developed in previous studies, but several simple alternatives are presented here in an attempt to find marketing strategies which many producers could use. Two proxies, or forecasts, for expected basis \((EB)\), as well as an indicator which combines the information obtained from both of these two proxies for expected basis, are presented. The first proxy assumes that an average of period 1 bases from previous years is a good predictor of this year’s period 1 basis. Historical Expected Basis \((HEB)\) is the average of period 1 bases for all years from 1974 up to year \(i\). In each year, the average of daily futures prices for the month of liquidation is subtracted from the average of daily cash prices for
Table 1. Marketing Strategies, Basis Indicators, and Expected Basis Proxies Considered

<table>
<thead>
<tr>
<th>Marketing Strategies</th>
<th>Description</th>
<th>Basis Indicators: ( k = 1/2 ) or 1 Chosen Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Routine <strong>CASH</strong></td>
<td>Sell cash commodity at harvest each year.</td>
<td>( CB &lt; (EB - \text{carry}<em>k \sigma</em>{EB}) ) <strong>SHORT HEDGING</strong></td>
</tr>
<tr>
<td>Routine <strong>SPECULATION</strong></td>
<td>Sell cash commodity and buy December, March, or May futures contract at harvest each year; sell contract in pre-expiration month.</td>
<td>Otherwise <strong>CASH</strong></td>
</tr>
<tr>
<td>Routine <strong>SHORT HEDGING</strong></td>
<td>Store cash commodity and sell December, March, or May futures contract at harvest; sell cash commodity and buy contract in pre-expiration month.</td>
<td><strong>SHORT HEDGING</strong> strategy based on basis indicator.</td>
</tr>
<tr>
<td>Routine <strong>STORAGE</strong></td>
<td>Store cash commodity at harvest; sell cash commodity in November, February, or April.</td>
<td>( CB &lt; (EB - \text{carry}<em>k \sigma</em>{EB}) ) <strong>SHORT HEDGING</strong></td>
</tr>
<tr>
<td>Basis Indicator: <strong>CASH</strong> or <strong>SHORT HEDGING</strong></td>
<td>Each year choose <strong>CASH</strong> or <strong>SHORT HEDGING</strong> strategy based on basis indicator.</td>
<td>Otherwise <strong>CASH</strong></td>
</tr>
</tbody>
</table>

**Expected Basis Proxies**

- \( EB = HCB \)  
- \( EB = HEB \)  
- \( EB = HCB \) and \( HEB \)  

- **HCB** = historical average of period 0 bases.  
- **HEB** = historical average of period 1 bases.  
- Basis Indicator for **SHORT HEDGING** must be satisfied with both \( EB = HCB \) and \( EB = HEB \).

That month. These monthly average period 1 bases are averaged together from 1974 to the year \( i \) to get an average period 1 basis. This average is used as a forecast of the year \( i \)'s period 1 basis. A second proxy assumes that the historical average of period 0 (harvest-time) bases (Historical Average of Current Bases, \( HCB \)) is a good indicator of whether the current basis will increase or decrease by period 1. If \( CB \) is greater than its historical average (\( HCB \)), it is more likely to decrease than increase from period 0 to period 1. Conversely, if \( CB \) is smaller than its historical average (\( HCB \)), it is more likely to increase than decrease. Thus, \( HCB \) can be used to represent \( EB \). As before, if \( CB \) is larger (smaller) than \( HCB, CB \) is larger (smaller) than \( EB \).

Neither of the two proxies provides a perfect forecast of basis, so each proxy has been adjusted by a measure of its variability. In Case I, each proxy was adjusted by its standard deviation. Thus, instead of \( CB < EB \) and \( CB \geq EB \), the adjusted indicators are \( CB < EB - \sigma \) and \( CB \geq EB + (\sigma \) denotes the standard deviation). In Case II, each proxy was adjusted by one-half standard deviation, that is, \( CB < EB - \sigma/2 \) and \( CB \geq EB + \sigma/2 \). The effect of these adjustments is to make a strategy other than **CASH** less likely to be used.

The strategies using the basis indicators are divided into three cases: using Historical Expected Basis only (\( HEB \)); using Historical Average of Current Bases Only (\( HCB \)); using both \( HEB \) and \( HCB \) (\( HCEB \)). In mathematical form, the net returns from each of the adjusted strategies using the first two indicators are as follows: if \( CB_i \geq EB_i + \sigma_E B_i \), then the net return for year \( i \) is as shown in equation (16), where \( \sigma_{EB_i} \) is the standard deviation of \( EB_i \); if \( CB_i < EB_i - \sigma_{EB_i} - CC_i \), then the net return in year \( i \) is as shown in equation (18). Substituting \( HEB \) or \( HCB \) for
Efficiency Criterion

$EB$ gives the net return of the strategy using $HEB$, or the net return of the strategy using $HCB$, respectively.

The third indicator makes the conditions for taking a futures position even more strict by requiring that both proxies give the same sign before any marketing method other than CASH is taken. If $CB$ is larger than or equal to not only $HEB$ but also $HCB$, then choose the CASH method. That is, if $CB_i \geq \max\{HEB_i + \sigma_{HEB_i}, (HCB_i + \sigma_{HCB_i})\}$, then the net return in year $i$ is as shown in equation (16). On the other hand, if $CB$ is smaller than both $HEB$ and $HCB$, then the SHORT HEDGING method is the best. Therefore, if $CB_i < \min\{HEB_i - \sigma_{HEB_i} - CC, (HCB_i - \sigma_{HCB_i})\}$, then the net return in year $i$ is as shown in equation (18).

**Procedures**

Cash and futures prices for hard red winter wheat were used to calculate daily basis between central Oklahoma and Kansas City for the period 1975–1990. Each year, June 20 basis ($CB$) was compared to the historical monthly basis for the month the underlying futures contract was liquidated ($HEB$), to the historical average daily basis on June 20 ($HCB$), and to both of them ($HCEB$). Daily bases were used for $CB$. Producers were assumed to make marketing decisions at harvest time (June 20) observing daily data. Monthly bases were used for expected basis ($EB$). In computing net returns, the closing cash and futures prices on June 20, as well as the first trading day in the month that the futures contract were liquidated, were used.

Carrying cost ($CC$) is defined as monthly storage cost plus monthly interest cost multiplied by the number of months wheat was stored. Assuming all wheat was stored in commercial storage, the monthly storage cost was assumed to be $2.5\text{¢}$ per bushel. Defining interest cost as the interest savings from paying off a loan, the production loan rate of a commercial bank was used as the interest rate in computing $CC$.

The three most actively-traded futures contracts (December, March, and May) were used in this analysis. As noted previously, cash wheat and futures contracts were assumed to be traded in two periods; at harvest time and when the futures contracts are liquidated. The futures contracts established at harvest were assumed to be liquidated on the first working day in either October, December, or March.

The net returns from using the three routine strategies and from choosing among the strategies using the basis indicators were computed for each year, 1975–1990. A total of 68 strategies were considered. Because some strategies had exactly the same return distributions, only 46 strategies with unique return distributions were analyzed. Each strategy is a set of net returns for 16 years. Table 1 shows summary statistics of the distributions of returns. These returns were evaluated using E-V, E-S, FSD, SSD, SDWRF, Mean-Gini, and MSTD.

The Generalized Stochastic Dominance Program (Cochran and Raskin) was used to perform the stochastic dominance analysis. Arrow-Pratt coefficients elicited at whole-farm income levels were adjusted to evaluate strategy choices described in terms of per bushel net returns. Risk aversion intervals $(-1.68, -0.01), (-0.01, 0.01), (0.01, 2.80)$, and $(2.80, 5.60)$ were used to represent risk-prefering, risk-neutral, slightly risk-averse, and strongly risk-averse decision makers, respectively. The ARA coefficients we used are based on those elicited for wheat farmers by King and Oamek (1983). The scales of the ARA coefficients were adjusted by
Table 2. Summary Statistics of Alternative Strategies Included in Efficient Sets

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Mean Dev. of Return ($/Bushel)</th>
<th>Std. of Return ($/Bushel)</th>
<th>Max. Return ($/Bushel)</th>
<th>Min. Return ($/Bushel)</th>
<th>Skewness Around Mean&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Skewness Around Target&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASH</td>
<td>3.1594</td>
<td>0.5458</td>
<td>4.00</td>
<td>2.15</td>
<td>-0.4935</td>
<td>-0.0267</td>
</tr>
<tr>
<td>STORAGE1</td>
<td>3.0519</td>
<td>0.4910</td>
<td>3.85</td>
<td>2.07</td>
<td>-0.1603</td>
<td>-0.4266</td>
</tr>
<tr>
<td>12LONG1</td>
<td>3.1163</td>
<td>0.5267</td>
<td>4.09</td>
<td>2.34</td>
<td>0.3022</td>
<td>-0.4816</td>
</tr>
<tr>
<td>12SHORT1</td>
<td>3.0950</td>
<td>0.5470</td>
<td>4.14</td>
<td>1.89</td>
<td>-0.4693</td>
<td>-0.0968</td>
</tr>
<tr>
<td>12HCB11</td>
<td>3.1282</td>
<td>0.5279</td>
<td>4.00</td>
<td>2.15</td>
<td>-0.4591</td>
<td>-0.0603</td>
</tr>
<tr>
<td>12HEB11</td>
<td>3.1362</td>
<td>0.5279</td>
<td>4.00</td>
<td>2.15</td>
<td>-0.4590</td>
<td>-0.0603</td>
</tr>
<tr>
<td>03LONG1</td>
<td>3.0827</td>
<td>0.6471</td>
<td>4.02</td>
<td>2.19</td>
<td>-0.0529</td>
<td>-0.3369</td>
</tr>
<tr>
<td>03SHORT3</td>
<td>3.0215</td>
<td>0.5853</td>
<td>3.88</td>
<td>1.85</td>
<td>-0.3202</td>
<td>-0.1629</td>
</tr>
<tr>
<td>03SHORT1</td>
<td>3.1285</td>
<td>0.7402</td>
<td>4.20</td>
<td>2.01</td>
<td>0.0187</td>
<td>-0.3883</td>
</tr>
<tr>
<td>03HCB31</td>
<td>3.1870</td>
<td>0.5687</td>
<td>4.00</td>
<td>2.15</td>
<td>-0.4701</td>
<td>-0.0229</td>
</tr>
<tr>
<td>05LONG2</td>
<td>3.0187</td>
<td>0.5795</td>
<td>3.97</td>
<td>1.94</td>
<td>-0.1742</td>
<td>-0.2820</td>
</tr>
<tr>
<td>05LONG3</td>
<td>2.9290</td>
<td>0.5275</td>
<td>3.78</td>
<td>2.00</td>
<td>0.0165</td>
<td>-0.6121</td>
</tr>
<tr>
<td>05LONG1</td>
<td>3.0705</td>
<td>0.6182</td>
<td>4.09</td>
<td>2.23</td>
<td>0.1143</td>
<td>-0.2820</td>
</tr>
<tr>
<td>05SHORT2</td>
<td>3.1353</td>
<td>0.6171</td>
<td>4.11</td>
<td>2.05</td>
<td>-0.1923</td>
<td>-0.0795</td>
</tr>
<tr>
<td>05SHORT3</td>
<td>3.0618</td>
<td>0.5911</td>
<td>4.05</td>
<td>1.96</td>
<td>-0.2159</td>
<td>-0.1412</td>
</tr>
<tr>
<td>05SHORT1</td>
<td>3.1407</td>
<td>0.7368</td>
<td>4.27</td>
<td>1.90</td>
<td>-0.0834</td>
<td>-0.4781</td>
</tr>
<tr>
<td>05HCB21</td>
<td>3.1952</td>
<td>0.5772</td>
<td>4.01</td>
<td>2.15</td>
<td>-0.4334</td>
<td>-0.0225</td>
</tr>
<tr>
<td>05HCB31</td>
<td>3.1994</td>
<td>0.5855</td>
<td>4.05</td>
<td>2.15</td>
<td>-0.3824</td>
<td>-0.0211</td>
</tr>
<tr>
<td>05HCB11</td>
<td>3.1987</td>
<td>0.6220</td>
<td>4.20</td>
<td>2.15</td>
<td>-0.1103</td>
<td>-0.1057</td>
</tr>
<tr>
<td>12HCB21</td>
<td>3.1946</td>
<td>0.5701</td>
<td>4.08</td>
<td>2.15</td>
<td>-0.4792</td>
<td>-0.0204</td>
</tr>
<tr>
<td>12HCB11</td>
<td>3.1530</td>
<td>0.5360</td>
<td>4.14</td>
<td>2.15</td>
<td>-0.4279</td>
<td>-0.0582</td>
</tr>
<tr>
<td>12HEB11</td>
<td>3.1260</td>
<td>0.5239</td>
<td>4.00</td>
<td>2.15</td>
<td>-0.4600</td>
<td>-0.0609</td>
</tr>
<tr>
<td>03HEB22</td>
<td>3.1845</td>
<td>0.5596</td>
<td>4.00</td>
<td>2.15</td>
<td>-0.5066</td>
<td>-0.0255</td>
</tr>
<tr>
<td>03HEB12</td>
<td>3.1702</td>
<td>0.5554</td>
<td>4.00</td>
<td>2.15</td>
<td>-0.4746</td>
<td>-0.0241</td>
</tr>
<tr>
<td>03HEB22</td>
<td>3.1887</td>
<td>0.5623</td>
<td>4.00</td>
<td>2.15</td>
<td>-0.5125</td>
<td>-0.0225</td>
</tr>
<tr>
<td>05HCB22</td>
<td>3.2046</td>
<td>0.5879</td>
<td>4.11</td>
<td>2.15</td>
<td>-0.3844</td>
<td>-0.0195</td>
</tr>
</tbody>
</table>

Note: The name of the strategy consists of four parts: the first two digits denoting the underlying contract (i.e., 03 is March, 05 is May, and 12 is December contract); a name of a routine strategy (CASH, STORAGE, SPEC, or SHORT) or basis indicator used (HCB, HEB, or HCEB); a digit denoting the period when the contract is liquidated (1 = October 1, 2 = December 1 of the current year, and 3 = March 1 of the following calendar year); at the end, the subscripts '1' denoting that the indicator has been adjusted by one standard deviation (Case I) and '2' denoting that the indicator has been adjusted by half standard deviation (Case II). For example, 12HCB11 is the strategy of choosing one of the two alternative marketing methods according to the HCB indicator adjusted by one standard deviation (Case I) in each year and using the December contract which is liquidated on October 1 of the current year.

<sup>a</sup> Skewness around the mean is measured as \((1/n)\sum(\pi - \mu)^3/\sigma^3\), where \(\mu\) is mean and \(\sigma\) is the standard deviation.

<sup>b</sup> Skewness around the target is measured as \((1/n)\sum(\pi - \tau)^3/s^3\), where \(\tau\) is the specified target return and \(s\) is the standard deviation around the target.

the unit of outcome scale following Raskin and Cochran (1986). The levels of \(\alpha\) and \(\beta\) in the MSTD criterion were obtained by substituting the values of ARA coefficients into equation (14). In this analysis, the cash prices at harvest have been used as the target levels of net returns.

Results

The strategies included in each efficiency rule are indicated in Table 2. Individual strategies are described in a footnote to Table 2. The E-V efficient set consists of 13 strategies (Table 3). In FSD, 19 of 46 strategies are undominated. The SSD
Table 3. Efficient Strategies with E-V, FSD, SSD, SDWRF, E-S, and M-G

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Range of ARA&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Efficient Set&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-V</td>
<td>(0, ∞)</td>
<td>CASH</td>
</tr>
<tr>
<td></td>
<td>05 HCB&lt;sub&gt;2&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>12 HEB&lt;sub&gt;1&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>FSD</td>
<td>(−∞, ∞)</td>
<td>STORAGE1</td>
</tr>
<tr>
<td></td>
<td>03 SHORT&lt;sub&gt;3&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>05 SHORT&lt;sub&gt;1&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>05 HCB&lt;sub&gt;3&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>SSD</td>
<td>(0, ∞)</td>
<td>12 LONG&lt;sub&gt;1&lt;/sub&gt;</td>
</tr>
<tr>
<td>SDWRF</td>
<td>(−1.68, −0.01)</td>
<td>03 SHORT&lt;sub&gt;1&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td>(−0.01, 0.01)</td>
<td>05 HCB&lt;sub&gt;2&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td>(0.01, 2.80)</td>
<td>12 LONG&lt;sub&gt;1&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td>(−1.68, 5.60)</td>
<td>03 SHORT&lt;sub&gt;1&lt;/sub&gt;,</td>
</tr>
<tr>
<td>E-S</td>
<td>(0, ∞)</td>
<td>12 HCB&lt;sub&gt;2&lt;/sub&gt;</td>
</tr>
<tr>
<td>M-G</td>
<td>(0, ∞)</td>
<td>12 HCB&lt;sub&gt;2&lt;/sub&gt;</td>
</tr>
</tbody>
</table>

<sup>a</sup> ARA is the Arrow-Pratt absolute risk aversion coefficient.

<sup>b</sup> See notes to Table 2 for names of strategies.

* These strategies do not appear in any smaller range of SDWRF (see footnote 10 for details).

Efficient set includes five strategies. The results of the SDWRF analysis show that one to four strategies are included for given ranges of ARA coefficients. Note that for the entire range (−1.68, 5.60), nine distinct strategies are included. Two strategies, 05 HCB<sub>3</sub> and 12 HCB<sub>1</sub>, which did not appear in any of the smaller ranges, are included in this entire range. Using E-S, two strategies are included in the efficient set. Both of them are also in the SSD set, which is consistent with Porter’s (1974) result. Two strategies are included in the M-G set; both are in the SSD set, which is consistent with results by Yitzhaki (1982), and by Buccola and Subaei (1984).

Twelve ranges of ARA coefficient were considered for evaluation under MSTD (see Table 4). The first three cases were for a DM who was risk preferring below target, the next three ranges were for a DM who was risk neutral below target, the following three ranges were for a DM who was risk averse below target, and the last three ranges were for a DM who was strongly risk averse below target. Each group consisted of three ranges of risk preferring, risk neutral, and risk averse above target. The interval of ARA coefficients below target was restricted to be greater than −0.17, and the interval of ARA coefficients above target to be smaller than 0.17 for α and β to be real numbers.

---

<sup>10</sup> In essence, given three ranges of ARA, A = (r<sub>1</sub>, r<sub>2</sub>), B = (r<sub>3</sub>, r<sub>4</sub>), and C = (r<sub>1</sub>, r<sub>4</sub>), where r<sub>1</sub> < r<sub>2</sub> < r<sub>3</sub> < r<sub>4</sub>. There exists some choice H which is undominated in range C. However, H can be dominated for smaller ranges of ARA, say A and B. Therefore, in this empirical example, SDWRF ignores some choices in smaller ranges.

<sup>11</sup> If α and β are obtained numerically (see footnote 7), the ranges of r<sub>1</sub> and r<sub>2</sub> may not have to be restricted.
Table 4. Efficient Strategies with the MSTD Criterion

<table>
<thead>
<tr>
<th>Absolute Risk Aversion Coefficient Intervals</th>
<th>Efficient Set</th>
<th>Stochastic Dominance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below Target ($r_1$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(-0.17, -0.01)$</td>
<td>$05HCB2_2$</td>
<td>FSD</td>
</tr>
<tr>
<td>Risk preferring</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(-0.01, 0.01)$</td>
<td>$05HCB2_2$</td>
<td>FSD</td>
</tr>
<tr>
<td>Risk neutral</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(-0.01, 0.06)$</td>
<td>$05HCB2_2$</td>
<td>FSD</td>
</tr>
<tr>
<td>Risk averse</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(-0.17, -0.01)$</td>
<td>$05HCB2_2$</td>
<td>FSD</td>
</tr>
<tr>
<td>Risk neutral</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(-0.01, 0.01)$</td>
<td>$05HCB2_2$</td>
<td>FSD</td>
</tr>
<tr>
<td>Risk neutral</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(0.01, 0.17)$</td>
<td>$05HCB2_2$</td>
<td>FSD(SSD)</td>
</tr>
<tr>
<td>Risk averse</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(2.80, 5.60)$</td>
<td>$05HCB2_2$</td>
<td>FSD</td>
</tr>
<tr>
<td>Strongly risk averse</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(2.80, 5.60)$</td>
<td>$05HCB2_2$</td>
<td>FSD(SSD)</td>
</tr>
<tr>
<td>Risk averse</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(-0.17, 5.60)$</td>
<td>$05HCB2_2$</td>
<td>FSD</td>
</tr>
</tbody>
</table>

The MSTD efficient set includes only one strategy $05HCB2_2$ for any given range of ARA coefficients (Table 4). The strategy selected under MSTD is the least negatively skewed (Table 2). The strategies selected by the MSTD criterion is also in the FSD set, which is consistent with Theorem 2. However, MSTD reduces the FSD efficient set more than 90%. The strategy selected by a DM who is risk averse both below and above the target is also included in the SSD efficient set in Table 3, which is consistent with Theorem 2. The efficient set under MSTD of an everywhere risk averse DM is smaller than the efficient set with SSD. The FSD efficient set includes five strategies, but the MSTD efficient set for everywhere risk averse DM includes one.

The last row in Table 4 provides a comparison with the last row of the SDWRF results in Table 3. Both efficiency measures consider the same range of ARA, between $-1.68$ and $5.60$. However, MSTD uses a different range of ARA for below target ($r_2 = (-1.68, 0.17)$) and above target ($r_1 = (-0.17, 5.60)$), while SDWRF does not allow such a separation. For the entire range of $r_2 = (-1.68, 0.17)$ and $r_1 = (-0.17, 5.60)$, MSTD contains one strategy. As SDWRF includes nine distinct strategies for the entire range of $(-1.68, 5.60)$ (Table 3), the efficient set of SDWRF is larger than that of MSTD. Moreover, two strategies which are not included in any of the smaller intervals in SDWRF appear in the entire range (Table 3).
IV. Conclusions

This paper develops a new risk efficiency model, Mean-Separated Target Deviations (MSTD). Conventional measures of risk do not distinguish between below-target and above-target outcomes, or else impose risk neutrality for above-target outcomes. The model is motivated by the intuition that although investors are comfortable with expected value as a measure of return, they respond in different ways to potential outcomes below a target return than to potential outcomes above a target return. Dispersion as a measure of risk is separated into two parts: below-target deviations and above-target deviations. The risk measure in the model is below-target deviations minus above-target deviations, each term is weighted by probability and investor's risk attitude. Separating above-target returns from below-target returns allows the model to be affected by skewness.

The MSTD model is shown to be congruent with von Neumann-Morgenstern expected utility theory. MSTD is a special case of first-degree stochastic dominance. The restrictions on the parameters necessary for MSTD to be a special case of second-degree and third-degree stochastic dominance rules are also derived. The disadvantage of MSTD is that it places more restrictions on an investor's utility function than some other criteria and therefore could yield inappropriate recommendations if the restrictions were inconsistent with a given investor's preferences. The MSTD is essentially based on a specific utility function and as applied here requires specifying a range for the investor's level of absolute risk aversion.

The MSTD model generalizes Fishburn's (1977) model. Fishburn's model assumes risk neutrality above the target. The MSTD model avoids such a restriction. The MSTD model differs from Holthausen's (1981) model in that it uses expected return as a measure of return as in Fishburn's model while Holthausen used above-target return as a measure of return. The MSTD model also goes beyond Fishburn and Holthausen by showing how MSTD can be used with an interval analysis similar to stochastic dominance with respect to a function.

Efficient sets were determined for alternative marketing strategies to evaluate the usefulness of MSTD. Results reveal that the MSTD efficient set contains one strategy among forty-six possible strategies for any given range of decision maker's absolute risk aversion coefficients and reduces the efficient set by more than 90% relative to FSD. For corresponding ranges of risk preferences, MSTD yields a smaller efficient set than stochastic dominance rules.

Appendix

Proof of Theorem 1. The basic idea of the proof follows that of Theorem 2 in Fishburn (1977). For notational simplicity, let \( t = 0 \) with \( u(0) = 0 \). For \( \pi < -1 \), let \( h \) exceed \(-\pi\) and consider two gambles: \( F_1 \) is the fifty-fifty gamble for \(-1\) or \( g > 0 \), and \( F_2 \) is the distribution which has probability \( \phi(1)/2\phi(-\pi) \) for \( \pi \) and probability \( [2\phi(-\pi) - \phi(1)]/2\phi(-\pi) \) for \( h \), where \( g = \phi(1)/\phi(-\pi) + h[2\phi(-\pi) - \phi(1)]/\phi(-\pi) + 1 \) and \( \phi(g) = \phi(h)[2\phi(-\pi) - \phi(1)]/\phi(-\pi) \). Then \( E(F_1) = E(F_2) \) and \( \Omega(F_1) = \Omega(F_2) \). Therefore, \((1/2)u(-1) + (1/2)u(g) = u(\pi)\phi(1)/2\phi(-\pi) + u(h)[2\phi(-\pi) - \phi(1)]/2\phi(-\pi) \). Solving for \( u(\pi) \) yields \( u(\pi) = \pi - 5\phi(-\pi) \).
For $-1 < \pi < 0$, let $h$ exceed $-\pi$ and consider two gambles: $F_1$ is the fifty-fifty gamble for $\pi$ or $h$, and $F_2$ is the distribution which has probability $\phi(-\pi)/2\phi(1)$ for $-1$ and probability $[2\phi(1) - \phi(-\pi)]/2\phi(1)$ for $g$, where $g > \phi(-\pi)/[2\phi(1) - \phi(-\pi)]$. Define $h = 2g - (1 + g)\phi(-\pi)/\phi(1)$ and $\theta(h) = \theta(g)[2\phi(1) - \phi(-\pi)]/\phi(1)$. Then $E(F_1) = E(F_2)$ and $\Omega(F_1) = \Omega(F_2)$. Therefore, $(1/2)u(\pi) + (1/2)u(h) = u(-1)\phi(-\pi)/2\phi(1) + u(g)[2\phi(1) - \phi(-\pi)]/2\phi(1)$. Solving for $u(\pi)$ yields $u(\pi) = \pi - \delta\phi(-\pi)$.

For $0 < \pi < 1$, consider two gambles: $F_1$ if fifty-fifty gamble for $\pi$ or $h$, where $h < -\pi$, and $F_2$ gives $1$ with probability $\theta(\pi)/\theta(1)$ and $g$ with $[\theta(1) - \theta(\pi)]/\theta(1)$, where $g > -\theta(\pi)/[\theta(1) - \theta(\pi)]$. Define $h = 2\theta(\pi)/\theta(1) + 2g - g\theta(\pi)/\theta(1) - \pi$ and $\phi(-h) = 2\phi(-g) - 2\phi(-g)\theta(\pi)/\theta(1) - \theta(\pi)\lambda/\delta$. Then $E(F_1) = E(F_2)$ and $\Omega(F_1) = \Omega(F_2)$ so that $(1/2)u(\pi) + (1/2)u(h) = u(1)\theta(\pi)/\theta(1) + u(g)[\theta(1) - \theta(\pi)]/\theta(1)$. Solving for $u(\pi)$ yields $u(\pi) = \pi + \lambda\theta(\pi)$. Therefore, proof is completed.

PROOF OF THEOREM 2. If $F$ dominates $G$ by FSD, then $F(\pi) \leq G(\pi)$ for all values of $\pi$, where $F(\pi)$ and $G(\pi)$ are the cumulative distribution functions of return on alternative risky actions $F$ and $G$, respectively.

$$
\Delta = SD_t(F) - SD_t(G)
$$

$$
\Delta = \delta \int_{-\infty}^{t} (t - \pi)^\alpha [dF(\pi) - dG(\pi)] \tag{19}
$$

$$
\quad - \lambda \int_{t}^{\infty} (\pi - t)^\beta [dF(\pi) - dG(\pi)].
$$

Integrating equation (19) by parts,

$$
\Delta = \alpha \delta \int_{-\infty}^{t} (t - \pi)^\alpha - 1 [F(\pi) - G(\pi)] d\pi
$$

$$
\quad + \beta \lambda \int_{t}^{\infty} (\pi - t)^\beta - 1 [F(\pi) - G(\pi)] d\pi, \tag{20}
$$

since $F(\infty) = G(\infty) = 1$ and $F(-\infty) = G(-\infty) = 0$. $\epsilon_1$ is the lower limit of integration, and $\delta$ and $\lambda$ are positive constants. By FSD, $F(\pi) \leq G(\pi)$ for all $\pi$. Therefore, equation (20) is nonpositive for $\alpha \geq 0$ and $\beta \geq 0$. If, for any probability density functions, $F$ dominates $G$ by FSD, then the mean of $F$ is at least as large as that of $G$ for $\alpha \geq 0$, and $\beta \geq 0$. Under the assumption that $F$ and $G$ differ in either mean or separated semivariance, the above result is sufficient to guarantee that $F$ dominates $G$ by the MSTD criterion.
Integrating (20) by parts,

\[
\Delta = \alpha (\alpha - 1) \delta \int_{-\infty}^{t} (t - \pi)^{\alpha - 2} [F_1(\pi) - G_1(\pi)] \, d\pi \\
+ \beta (\epsilon_2 - t)^{\beta - 1} \lambda [F_1(\infty) - G_1(\infty)] \\
- \beta (\beta - 1) \lambda \int_{t}^{\infty} (\pi - t)^{\beta - 2} [F_1(\pi) - G_1(\pi)] \, d\pi,
\]

(21)
since \( F_1(-\infty) = G_1(-\infty) = 0 \). \( \epsilon_2 \) is the upper limit of integrations. \( F_1(\pi) \leq G_1(\pi) \) for all \( \pi \) by SSD. Therefore, equation (21) is nonpositive and thus SSD implies MSTD for \( \alpha \geq 1 \) and \( 0 \leq \beta \leq 1 \).

Integrating (21) by parts,

\[
\Delta = \alpha (\alpha - 1)(\alpha - 2) \delta \int_{-\infty}^{t} (t - \pi)^{\alpha - 3} [F_2(\pi) - G_2(\pi)] \, d\pi \\
+ \beta (\epsilon_2 - t)^{\beta - 1} \lambda [F_1(\infty) - G_1(\infty)] \\
- \beta (\beta - 1)(\epsilon_2 - t)^{\beta - 2} \lambda [F_2(\infty) - G_2(\infty)] \\
+ \beta (\beta - 1)(\beta - 2) \lambda \int_{t}^{\infty} (\pi - t)^{\beta - 3} [F_2(\pi) - G_2(\pi)] \, d\pi.
\]

(22)
since \( F_2(-\infty) = G_2(-\infty) = 0 \). \( F_2(\pi) \leq G_2(\pi) \) for all \( \pi \) by TSD. Therefore, equation (22) is nonpositive and thus TSD implies MSTD for \( \alpha \geq 2 \) and \( 0 \leq \beta \leq 1 \). ■

References


