Optimal frequency and quantity of agricultural lime applications

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Abstract

Crop yields can be limited by soil pH. This study was conducted to determine the optimal level and frequency of lime applications to low pH soils. A model was developed to reflect the dynamics of pH change in response to lime application and continuous cropping. Data were obtained from field trials. A wheat (Triticum aestivum) grain yield response to soil pH function was estimated. An evolutionary algorithm was used to solve the discontinuous nonlinear model to determine the optimal level and frequency of lime application. Optimal lime application strategies that maximize net present value of returns from continuous monoculture wheat production were determined for several levels of initial soil pH.

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1. Introduction

Crop yields can be greatly influenced by soil pH. The pH of the soil plant-root zone influences the ability of plants to acquire essential nutrients from the soil. If the soil pH declines below a critical level, the solubility of aluminum and manganese ions increases, resulting in toxicity and lower yields. Soil acidity affects plant growth in several ways. Toxicity, caused by increased mobility of soil aluminum, is thought to be the most serious of these effects (Black, 1992). Aluminum is the most abundant metal in soils and becomes readily soluble when the pH drops below 5.5. At pH 4.0,
the cation exchange complex of soils is completely saturated with aluminum ions, which results in plants being deprived of essential cations (Foth and Ellis, 1997).

Acidification of soils over time is a consequence of the removal of base elements with harvested crops and the application of nitrogen fertilizers. Harvested crops contain base elements such as potassium, magnesium, and calcium. For example, a harvest of one Mg of wheat (*Triticum aestivum*) grain removes base elements in the amount that is equivalent to 11–14 kg of calcium. Although this quantity would not appreciably change soil pH in one year, continuous cropping and removal of grain will over time increase soil acidity of the plant root zone. Application of nitrogen fertilizers affects soil acidity through microbial activity of soil microorganisms. Soil microbes convert ammonia cations (base) into nitrates (acid), a process that releases hydrogen cations (acid) as byproducts (Paul and Clark, 1996).

In a 1940 US Department of Agriculture bulletin, Shorey (1940) presented a “lime line” drawn near the geographical center of the continental United States. At the time, soil acidity was not a problem for most cropland soils to the west of the lime line that included most of the Great Plains. These soils were characterized by Shorey as “lime accumulating”. Hence, for many years land in the southern Great Plains was cropped continuously with little concern for soil pH. However, a 1985 survey of Oklahoma fields cropped continuously to winter wheat found that more than 30% of 17,000 samples had pH less than 5.5. A similar survey in 1996 found that 39% of samples had a pH less than 5.5 (Zhang et al., 1998). These soil-testing programs were not conducted for the purpose of drawing statistical inferences regarding changes in pH over time or proportion of the state’s cropland with various pH levels. However, these data suggest that acidity levels in many wheat fields in the region may be sufficiently high to limit grain yield and that the number of fields for which acidity is a problem has increased over time. Wheat grain yield response to soil pH has not been precisely defined for the region.

Agricultural lime is a soil amendment that may be applied to increase soil pH. The term “agricultural lime” is usually applied to any form of liming materials that contain calcium or magnesium oxides, hydroxides or carbonates and can be used in neutralizing soil acidity (Shorey, 1940). Field experiments have demonstrated that lime application changes the soil pH over time and helps to remove negative effects of soil acidity (Coventry et al., 1997; Krenzer and Westerman, 1993; Malhi et al., 1983). Lime does not contain primary nutrients and is classified as a soil amendment rather than a fertilizer. Unlike many fertilizers, lime has a strong carryover effect. The economics of liming for cropland west of the historical lime line have not been determined. Precise specification of wheat grain yield response to soil pH in the region has not been done.

The objective of this study is to determine soil pH change over time in response to cropping, soil pH change over time in response to application of agricultural lime, wheat yield response to soil pH, and the economics of lime application to soils used to produce winter wheat in the southern Great Plains of the United States. Since it is soil pH that affects crop production and liming is a tool to regulate soil pH, first the effect of lime application on soil pH is investigated, second the effect of pH on yields, and finally the economics of lime application.
2. Materials and methods

2.1. Soil pH change over time

Black (1992) cites a class of the half-life, or decay, models that have been extensively exploited to study residual effects of fertilizers. The half-life concept assumes that the rate of change in the level of some factor, \( x \), is proportional to the difference between \( x \) and an equilibrium level, \( x_{eq} \). This concept is described by a negative exponential function:

\[
x_t = x_{eq} + (x_{init} - x_{eq})e^{-ct}
\]

where \( x_{init} \) is an initial level of \( x \) and \( c \) is a loss constant (Black, 1992). The level of \( x \) asymptotically approaches \( x_{eq} \) as time passes. A transformed version of (1) was used by Bromfield et al. (1983) to model soil acidification over time in Australian pastures. Their model is:

\[
\text{pH}_t = \text{pH}_0 + \frac{k}{C_0} e^{-ct/C_1}
\]

where \( k \) and \( c \) are constants related to soil types (notation has been changed slightly). The model does not consider the effect of lime applications on soil acidity.

An analogous form of (2) can be borrowed from studies dealing with modeling of biological systems (Hannon and Ruth, 1998):

\[
x_{t+1} = x_t + k x_t \left(1 - \frac{x_t}{x_{crit}}\right)
\]

Eq. (3) is a discrete representation of the logistic function (Hannon and Ruth, 1998). As depicted in Eq. (3), \( x \) approaches \( x_{crit} \) asymptotically as \( t \) approaches infinity. The constant \( k \) reflects a rate of adjustment from the initial state to the final or critical state (\( x_{crit} \)). For our purposes Eq. (3) may be modified to obtain:

\[
\text{pH}_{t+1} = \text{pH}_t + k \text{pH}_t \left(1 - \frac{\text{pH}_t}{\text{pH}_{crit}}\right)
\]

Eq. (4), with hypothetical parameter values of 0.03 for \( k \) and 4.2 for \( \text{pH}_{crit} \) and an initial \( \text{pH} \) of 5.3, is graphed in Fig. 1. The chart depicts the hypothesized gradual decline in \( \text{pH} \) over time expected to occur in continuously cropped fields.

2.2. Soil pH change after lime application

Change in soil \( \text{pH} \) over time in response to lime application depends upon the soil type, lime rate and lime quality (Foth and Ellis, 1997). Foth and Ellis (1997) describe soil \( \text{pH} \) change over time due to liming as an increase following application and gradual decrease with time. Other researchers have described the difficulty associated
with attempts to quantify the change in pH after lime application (Black, 1992; Coventry et al., 1997; Malhi et al., 1983). Lime application has a long-term impact, often more than 20 years that in general exceeds the life of the experiment. Bongiovanni and Lowenberg-DeBoer (1998) tested a lime carryover formula adapted from Black to model pH change over time:

\[
pH_t = B_s \left( \sqrt{\frac{LA - \sum LC_t + 1 - 1}{C_0}} \right) + pH_0
\]

Fig. 1. Hypothetical change in pH over time resulting from continuous cropping to wheat based upon the following function: \( pH_{t+1} = pH_t + k \left( pH_t - pH_{crit} \right) \) where \( pH_0 = 5.3, k = 0.03, \) and \( pH_{crit} = 4.2 \).

where soil pH at year \( t \) is a function of initial soil pH, \( pH_0 \), lime application rate, \( LA \), crop consumption of base elements, \( LC \), and soil buffering capacity, \( B_s \). The equation assumes an immediate increase in soil pH due to lime application and a continuous decrease over time due to removal of base elements with the harvested crop. Bongiovanni and Lowenberg-DeBoer (1998) found that this specification overestimated soil pH in the initial years after lime application and failed to adequately represent soil pH change after lime application.

Data from most lime application studies suggest that the carryover effect of liming should accommodate for an initial increase followed by a gradual decrease in soil pH (e.g., see Figs. 8-9 in Black, 1992). In the experiment reported by Black, the soil pH continued to increase for 5 years after lime application, and began decreasing in the sixth year. No prior modeling efforts have accounted for that type of a dynamic process. After considering several functional forms, it was determined that the process described by Black (1992) and others can be expressed by the following Eq. (6) that combines an exponential increase with an exponential decay:

\[
x_t = x_{init} + bt^e e^{bt}
\]
where \( x_t \) is the level of \( x \) at time \( t \), \( x_{\text{init}} \) is the initial level of \( x \), \( b \) is a parameter that defines a magnitude of an increase, and \( \alpha \) and \( \beta \) are parameters related to slopes of increase and decrease. This equation can be used to model the dynamics of soil pH change after lime application. The parameter \( b \) relates to lime rates and the parameters \( \alpha \) and \( \beta \) relate to soils. If data are available, these parameters can be estimated using nonlinear least-squares procedures. For our purpose Eq. (6) can be rewritten as

\[
\text{pH}_t = \text{pH}_{\text{init}} + b \alpha e^\beta t
\]  

(7)

where \( \text{pH}_t \) is the level of pH at time \( t \) and \( \text{pH}_{\text{init}} \) is the initial pH level.

Eq. (7) with an initial pH level of 4.6, and hypothetical parameter values of 0.4 for \( b \), 0.7 for \( \alpha \), and \(-0.11\) for \( \beta \) is graphed in Fig. 2. The chart confirms that the equation does display an exponential increase followed by an exponential decay and is consistent with the change in soil pH after lime application over time as described by Black (1992).

After a period of time following lime application it is reasonable to expect the decline in pH as reflected in Eq. (7) to be similar to the decline in pH as reflected in Eq. (5). In Fig. 3 both the hypothetical change in pH from cropping based upon Eq. (5), and the hypothetical change in pH after lime application based upon Eq. (7) are charted. The parameter estimates used to prepare Fig. 3 are the same as those used to prepare Figs. 1 and 2. However, in Fig. 3 the graph of Eq. (7) begins with the

Fig. 2. Hypothetical change in pH over time after lime application based upon the following function: \( \text{pH}_t = \text{pH}_{\text{init}} + b \alpha e^\beta t \) where \( \text{pH}_{\text{init}} = 4.6, \ b = 0.4, \ t = \) time in years, \( \alpha = 0.7, \ e = \) the number \( e \), and \( \beta = -0.11 \).
twelfth year after lime application \((t_0 = 12)\). For this set of hypothetical parameter values the two equations produce similar estimates of pH over time.

### 2.3. Wheat yield response to soil pH

There is extensive literature on modeling crop response to various agronomic inputs (Ackello-Ogutu et al., 1985; Burt, 1995; Hall, 1983; Frank et al., 1990; Berck and Helfand, 1990; Spillman, 1933). A number of functional forms have been used to model crop response. Some functions are easier to estimate (polynomial, logarithmic). Others are posited as being more consistent with plant response (von Liebig, Mitcherlich-Baule). Frank et al. (1990) performed a series of non-nested tests to model corn yield response to two inputs and concluded that data favored plateau growth functions. Paris and Knapp (1989) provided a computational means to estimate plateau crop response functions, therefore preserving the biological relationship between nutrients and yields (Paris and Knapp, 1989).

In terms of crop physiology, crop production response to some input factor is observed when the factor is limiting. This concept is described as a plateau response function that can be summarized as follows:

\[
y = \min[f_1(Z_1, u_1), f_2(Z_2, u_2), \ldots, f_n(Z_n, u_n)]
\]

where \(Z_1, \ldots, Z_n\) are inputs, \(u_1, \ldots, u_n\) are disturbances associated with each input, and \(f_1, \ldots, f_n\) are functions of responses to the inputs (Paris and Knapp, 1989).
For specifying the response function for a single variable input, Eq. (8) can be written as:

\[
y = \min\{ f(Z, u_z), M \}
\]  

(9)

where \( M \) is a yield at the level where \( Z \) is no longer a limiting factor. Here, \( M \) is a function of \( n-1 \) factors in (8) and represents a random variable related to factors that cannot be controlled in a given experiment. Relative to input \( Z \), \( M \) is a constant, and assuming the normality of disturbances around \( M \), there should be observations above and below \( M \).

Many experiments include treatments in which inputs under investigation are applied in excessive amounts. According to (9), yields from treatments with excess quantity of a controllable input are no longer a function of that input, because it is not limiting. Fitting a regular quadratic response function will associate the yields beyond the maximum point with \( Z \), whereas (9) suggests that after some threshold level of \( Z \), this factor is not limiting and does not affect the crop yield. Observations below \( M \) would suggest a negative effect of excess of \( Z \) and observations above \( M \) would shift the maximum of a quadratic function rightward. These arguments explain the observed overestimation of optimal input levels when non-plateau functions were fitted (Frank et al., 1990; Hall, 1983).

Hall (1983) used a simplified functional relationship between lime rates and yields of alfalfa, corn, and soybeans. Hall (1983) estimated the optimal values of lime application based on one year's data, and arbitrarily assumed the longevity of the lime effect as 5 years. Based on those assumptions he fitted several response functions [linear-response plateau (LRP), logarithmic, power, quadratic-response plateau, and square root] and concluded that plateau functions avoid extremely large optimal rates. His findings regarding specification of a functional form are consistent with those of Paris and Knapp (1989) and Frank et al. (1990).

Hall (1983) suggested that least squares estimation criteria would not prohibit large prediction errors at or near economic optimum if they would be offset by small errors elsewhere. Plateau functions disregard errors above the maximum. Hence, estimates of optimal rates often become more reasonable and practical.

Mahler and McDole (1987) estimated quadratic and LRP wheat response to soil pH functions for artificially acidified soil in northern Idaho in the western United States. They found that the LRP model provided a better representation of the data and described the spline point as the minimum acceptable pH value. They concluded that a pH of 5.19–5.37 was required to achieve the wheat yield plateau (Mahler and McDole, 1987).

2.4. Data for estimating soil pH change over time

Eq. (4) can be used to describe the gradual change in pH over time resulting from continuous cropping with no lime application. Experimental data were not available to estimate parameters \( k \) and \( \text{pH}_{\text{crit}} \). Westerman (1987) observed a decrease in soil pH of 1.5–2.5 units over a period of more than 20 years for soils in the region.
Assuming that the soil pH has declined by 2–2.5 units during 25–30 years of continuous cropping from an initial pH level of 6.5–7.0, and that the critical soil pH level cannot drop below pH 3.9 (the point at which aluminum concentration reaches toxic levels), one can calibrate the parameter $k$ as 0.03, and $\text{pH}_{\text{crit}}$ as 3.9. This results in an approximate decline of 1.5 pH units during the first 10 years, followed by an additional decline of 0.5 units during the second 10 years.

2.5. Data for estimating soil pH change after lime application

Data from a lime rate experiment were used to estimate changes in soil pH after liming. The field experiment plots were located in a farm field near Carrier, Oklahoma in the southern Great Plains of the United States. The Pond Creek silt loam soil had been continuous cropped for a number of years and had never received a lime application. Over time the soil acidified. The average pH level of the field was determined to be 4.6 in 1978. Individual plot measures of pH were not taken prior to the lime application.

The lime rate that would increase the average soil pH of the field to near neutrality (pH of 6.5–7.0), $X$, was estimated by the researchers to be 10.8 Mg ha$^{-1}$ of effective calcium carbonate equivalent (ECCE). Lime rate treatments used in the experiment were derivatives of this recommended rate and referred to as 0, 1/4X, 1/2X, X, and 2X, respectively. A relatively uniform section of the field was located and treatments of 0, 2.7, 5.4, 10.8, and 21.5 Mg ha$^{-1}$ of ECCE lime were replicated four times in small plots in 1978. Soil samples were taken from each of the 20 individual plots in 1979, 1980, 1981, 1982, 1983, and 1986. Hence, pH measures were available for four replications of five treatments for six seasons. Measures of soil pH were not taken in 1984 and 1985, and the experiment was discontinued after the 1986 season. A total of 120 observations were available to estimate the impact of a single lime application on soil pH change. All other nutrients, including nitrogen and phosphorus, were applied at the same rates across all plots. The levels were sufficient to ensure that yield was not limited as a result of these nutrients.

2.6. Data for estimating wheat yield response to soil pH

Wheat grain yield response to soil pH data were available from the experiment described in the previous section. In four of the wheat production seasons (1979, 1980, 1981, 1982), wheat grain yield data and soil pH were recorded from each of the four replications for each of the five treatments. Thus, 80 observations were available for estimating wheat grain yield response to soil pH.

2.7. Economic optimization

From an economic perspective, liming is a capital investment rather than an operating input because of its long-term effect. In previous research, the economic optimization was modeled under the limiting assumption that only a single application of lime could be made at the beginning of a fixed time period. This single
application model ignores several issues. It does not determine the optimal frequency of lime application. It also does not consider that the potential for applying additional lime in subsequent years may influence the optimal lime rate in the initial year. In this section a less restrictive model is formulated that enables simultaneous determination of the optimal application in year one and the optimal frequency and quantity of subsequent lime applications.

The objective function is:

\[
\max_{L_{R_t}} \text{NPV} = \sum_{t=1}^{T} \frac{P_G G_t(pH_t) - r_L L_{R_t} - CA_t}{(1 + i)^t}
\]

where NPV is the present value of returns ($\text{ha}^{-1}$) net of the cost of lime application over T years; \( p_G \) is the price of wheat ($\text{kg}^{-1}$); \( G_t \) is the grain yield (kg \text{ha}^{-1}) at year \( t \); grain yield at year \( t \) is a function of soil pH at year \( t \); \( r_L \) is the cost of lime ($\text{Mg}^{-1}$) in terms of ECCE; \( CA_t \) is lime application cost in year \( t \); and \( i \) is the discount rate. The choice variables are \( L_{R_t} \), lime application rates at year \( t \), in Mg of ECCE ha\(^{-1}\).

For a multiple application model the total effect of lime application accumulates effects of all previous applications. Depending on whether lime was applied in any particular year or not the change in soil pH, \( \Delta pH_t \), can be represented by Eq. (11):

\[
\Delta pH_t = d_1 b_1 e^{\beta t} + d_2 b_2 (t-1)^\alpha e^{\beta (t-1)} + \ldots + d_k b_k (t-k-1)^\alpha e^{\beta (t-k-1)} + \ldots + d_T b_T e^\beta
\]

or,

\[
\Delta pH_t = \sum_{k=1}^{t} d_k b_k (t-k-1)^\alpha e^{\beta (t-k-1)}
\]

\[
t = 1, 2, \ldots, T;
\]

\[
k = 1, 2, \ldots, t.
\]

The parameter \( d_k \) is a binary variable that equals one when the decision is to apply lime and zero otherwise.

The specified model has a discontinuous, nonlinear objective function, which makes it difficult to solve using traditional optimization techniques that require twice-differentiable objective functions. Dynamic programming could be a possible approach to this specific problem; however, the curse of dimensionality due to the interaction of liming effects in different periods of time complicates the search over the domain of choice variables.

Optimization methods based on evolutionary algorithms have been developed for non-smooth, multi-dimensional, or discontinuous objective functions (Mayer et al., 1996). Evolutionary algorithm is a term used to describe a class of computational models that attempt to mimic natural evolution to solve optimization problems.
Several evolutionary methods have been proposed including genetic algorithms, evolutionary programming, and evolution strategies. Mayer et al. (1996) demonstrated that genetic algorithms escape local optima to find the global optimum in case of “ill-behaved” objective functions. They used various optimization methods, including quasi-Newton, direct search, genetic algorithm, and simulated annealing, to solve a problem with an objective function that consisted of the combination of 48 factors. They concluded that simulated annealing and the genetic algorithm performed better in search of the global optimum. The quasi-Newton method with randomly selected initial values converged to the optimum in only 31% of re-runs.

The genetic algorithm applies the basic concept of the theory of natural selection—survival of the fittest. The method converts independent variables in the feasible region into sets of randomly chosen starting points. The values of each set are converted into a binary string, called a “chromosome”. Each “chromosome” represents a unique combination of the choice variables. A set of chromosomes forms a “parent” or initial population. The fitness of the population to the objective is estimated and those “chromosomes” are allowed to “cross-breed” based on their relative goodness of fit. The crossover, or “gene exchange,” occurs randomly, and the fitness of the “children’s” population is tested against the previous one. Successful combinations have more probability to “reproduce” and over time, the population converges to a single point or a number of near-optimal solutions (Mayer et al., 1999).

Evolutionary algorithms use numerical optimization methods (Michalewicz, 1996). They use the concept of “mutation,” or generating random numbers from a multivariate normal ($\mu$, $\lambda$) where $\mu$ is a vector of initial randomly chosen independent variables, and $\lambda$ represents a vector of self-adapting step sizes. The selection process replaces the least fit members from the population; the process is continued until $\lambda$ converges to zero. A detailed description and comparison of algorithms can be found in Michalewicz (1996). One shortcoming of the evolutionary techniques is that they do not have the concept of the optimal condition. The selection for the best solution is made only by testing against alternative solutions. These methods are more appropriate in situations when it is difficult or impossible to test the first order conditions.

3. Results

3.1. Estimation of soil pH change after lime application

The change in soil pH over time in response to lime application as characterized by Eq. (7) was specified. To account for different lime rates, Eq. (7) was modified for parameter estimation as:

$$\text{pH}_{it} = \text{pH}_{init} + \sum_{i=1}^{n} D_i b_i t^p e^{\beta t}$$ (12)
where \( D_i \) is a dummy variable for the \( i \)th lime rate. The parameters \( \text{pH}_{\text{init}}, b_i, \alpha, \) and \( \beta \) were estimated using the SAS NLIN procedure (SAS Version 6.12). Parameter estimates are reported in Table 1. All estimates except for \( \beta \), the parameter used to describe the decrease in soil pH, are significant at the 0.01 level of probability. The parameter estimate for \( \beta \) is significant at the 0.05 level of probability. Based upon the estimated function, soil pH is expected to increase in each of the first seven years after lime application. However, measures of pH were only available for eight years after lime application. Soil pH was estimated to increase to 5.66, 6.10, 6.89, and 7.15 for treatments of 2.7, 5.4, 10.8, and 21.5 Mg ha\(^{-1}\) of ECCE lime, respectively. The fitted function estimates that the pH increased to 6.89 on the plots that received 10.8 Mg ha\(^{-1}\) of ECCE lime. This treatment (the X level of lime application) was expected to increase pH to within a range from 6.5 to 7.0 and was successful in doing so.

The expected change in pH over time resulting from various levels of lime application, and the observed pH values, are depicted in Fig. 4. It is clear that for this experiment, soil pH continued to increase for a number of years after lime application. Unfortunately, since the experiment was discontinued, it is not clear how long the benefits of liming persisted.

### 3.2. Estimation of wheat yield response to soil pH

In this study, it is assumed that wheat grain yield response can be represented as a function of soil pH. At the same time, pH is a function of lime rate and time, which results in the specification:

\[
Y = f(\text{pH}(LR, t)) \tag{13}
\]

where \( Y \) is wheat grain yield, pH is soil pH, LR is lime rate, and \( t \) is time.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Lime rates (X = 10.8 Mg ha(^{-1}))</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_i )</td>
<td>( \text{Control} )</td>
<td>0.3455**</td>
<td>0.5582**</td>
<td>0.9439**</td>
<td>1.0672**</td>
</tr>
<tr>
<td></td>
<td>( 1/4X )</td>
<td>(3.54)</td>
<td>(4.87)</td>
<td>(6.25)</td>
<td>(6.52)</td>
</tr>
<tr>
<td>pH(_{\text{init}})</td>
<td>( \text{Control} )</td>
<td>4.9510**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 1/4X )</td>
<td>(52.50)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha )</td>
<td>( \text{Control} )</td>
<td>0.7662**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 1/4X )</td>
<td>(3.28)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>( \text{Control} )</td>
<td>-0.1098*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 1/4X )</td>
<td>(-1.96)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Predicted pH(_{\text{max}})^b</td>
<td>( \text{Control} )</td>
<td>5.66</td>
<td>6.10</td>
<td>6.89</td>
<td>7.15</td>
</tr>
</tbody>
</table>

a Values in parentheses are asymptotic \( t \)-statistics.

b Predicted maximum pH achieved over time after liming. For the estimated function the predicted pH level increased for the first 7 years after application for each application level.

* Significant at the 0.05 probability level.

** Significant at the 0.01 probability level.
Since the lime effect on crops is spread over time, it is important to consider yield variability among years. Burt (1995) discussed the structure of the error terms for a nonlinear model specification. He pointed out the existence of large correlation among years in pooling long-term experiments. He suggested a multiplicative heteroskedasticity form of disturbances associated with year effects. Since the lime data are repeated measures over time, a test of random factors such as time and lime rates was conducted. The hypothesis is that years may introduce additional variability into the data, which would reduce the efficiency of parameter estimates. If yield is specified as:

$$y_{it} = \mu + \lambda_i + \tau_t + \varepsilon_{it}$$  \hspace{1cm} (14)
where $y_{it}$ is yield from the $i$th lime treatment in year $t$; $\lambda_i$ is a fixed effect of the $i$th lime rate; $\tau_t$ is a random effect of time, $\tau_t$ is independently and identically distributed as $N(0, \sigma^2_t)$; and $\varepsilon_{it}$ is the error term, then $y_{it}$ is independently and identically distributed as $N(\mu + \lambda_i, \Sigma)$, where:

$$
\Sigma = \begin{bmatrix}
\sigma^2_{11} + \sigma^2 & \cdots & \sigma^2 & \cdots & \cdots \\
\cdots & \sigma^2_{22} + \sigma^2 & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \sigma^2_{tt} + \sigma^2
\end{bmatrix}.
$$

(15)

Tests for the first-order autoregressive process and multiplicative and groupwise heteroskedasticity due to the time factor were conducted using the SAS 6.12 MIXED procedure.

Three different response models were estimated. A quadratic response model was estimated using the SAS 6.12 MIXED procedure using the REPEATED statement with the GROUP = REPLICATION$\times$LIME option. LRP and quadratic-plateau models were estimated using the SAS 6.12 NLIN procedure (SAS Institute, 1988). The parameter estimates of Eq. (10) were corrected for heteroskedasticity using the estimated generalized nonlinear least squares method. The following procedure was used. First, residuals from the nonlinear least squares estimation were obtained. Second, separate estimates of $\sigma^2_t$ were determined for each $t$. Third, the independent and dependent variables and the intercept and plateau parameters of the model were weighted by $\sigma^2_t$. These three steps were repeated until the variance of the residuals converged.

Parameter estimates for the wheat grain yield response to soil pH are reported in Table 2. The quadratic, LRP and quadratic-plateau models were compared using the non-nested hypothesis $J$-test, where predicted values from one model were used as an explanatory variable in another model (Greene, 1997). The results of the tests were inconclusive. No model was rejected in favor of an alternative based on the $J$-test.

Table 2
Parameter estimates of wheat grain response to soil pH$^a$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Quadratic</th>
<th>Linear-plateau</th>
<th>Quadratic-plateau</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>$-4875^{**}$</td>
<td>$-402^{**}$</td>
<td>$-10,750$</td>
</tr>
<tr>
<td>pH</td>
<td>$2580^{**}$</td>
<td>$638^{**}$</td>
<td>$(4,905$</td>
</tr>
<tr>
<td>pH$^2$</td>
<td>$-215^{**}$</td>
<td>$-437$</td>
<td>$-1.52$</td>
</tr>
<tr>
<td>$Y_{max}^{b}$</td>
<td>$3064$</td>
<td>$2955$</td>
<td>$3000$</td>
</tr>
<tr>
<td>Pred. pH$_{max}$</td>
<td>$6.16$</td>
<td>$5.26$</td>
<td>$5.61$</td>
</tr>
</tbody>
</table>

$^a$ The dependent variable is wheat grain yield (kg ha$^{-1}$). Values in parentheses are asymptotic $t$-statistics.

$^b$ $Y_{max}$ for the quadratic model is reported as the maximum point of the function.

** Significant at the 0.01 probability level.
Estimated values of the soil pH that produce maximum response of wheat grain yields are different for the models. As expected, the quadratic model resulted in the highest pH$_{\text{max}}$. Based upon the quadratic model, a soil pH level of 6.16 would be required to obtain the maximum expected yield of 3064 kg ha$^{-1}$. The LRP model estimated with weighted least squares indicates that a pH level of 5.26 is sufficient to obtain the maximum expected yield of 2955 kg ha$^{-1}$. Plateau functions produced estimates that were more rational from the agronomic point of view. Also, the soil pH level of 5.26 estimated for the plateau yield with the LRP model is within the range of 5.19–5.37 as reported by Mahler and McDole (1987) for the wheat response to pH study conducted in the western United States. The estimated LRP model and the observed grain yield and pH values, are presented in Fig. 5. The parameters estimated for the LRP model were used for economic analysis.

3.3. Optimal lime management

Eq. (10) was solved to determine optimal lime rates and optimal timing of lime applications using the standard evolutionary algorithm (Frontline Systems, 1999) available as an add-in package to the Microsoft Excel software. The model was specified to accommodate the lime carryover effect from possible applications in years 1 to 25. The model was based upon the assumption that the land would be used to produce continuous monoculture wheat.

The evolutionary solver algorithm was used to estimate optimal lime rates for four different levels of initial soil pH (4.1; 4.4; 4.8; 5.2). When multiple applications are
allowed, the number of dimensions in the dynamic model increases exponentially. To restrict the search domain, constraints were added to the optimization problem. The first application rate was restricted to a range between 0 and 11 Mg ha\(^{-1}\) and subsequent applications were restricted to be equal to each other with the maximum rate of 2.24 Mg ha\(^{-1}\). The evolutionary solver was executed several times with different sets of starting values to verify that the global solution was found. Results for four initial pH levels are presented in Table 3.

For an initial soil pH level of 4.1, if no lime is applied, the net present value of the land for the 25-year time horizon is estimated to be $2852 ha\(^{-1}\). However, if the optimal lime strategy is followed, the net present value increases by $679 to $3531 ha\(^{-1}\). For this low level of pH (4.1) the optimal lime application in the first year is estimated to be 6.38 Mg ha\(^{-1}\). Subsequent applications of 0.67 Mg ha\(^{-1}\) would be optimally applied in years 10, 13, 16, 19, 22, and 25.

The difference in net present value between the no lime and the lime strategies is estimated to be $579, $427, and $257 ha\(^{-1}\) for soil with initial pH levels of 4.4, 4.8, and 5.2, respectively. For an initial soil pH of 4.4, the estimated optimal strategy is to apply 5.04 Mg ha\(^{-1}\) in year one followed by applications of 1.08 Mg ha\(^{-1}\) in years 11, 16, and 21. If the initial soil pH is 4.8, the estimated optimal strategy is to apply 2.91 Mg ha\(^{-1}\) in year one followed by applications of 1.34 Mg ha\(^{-1}\) in year 10 and year 17. This situation is graphed in Fig. 6. The chart in Fig. 6 includes the estimated soil pH over the 25-year time horizon as well as the timing and level of lime application.

4. Discussion

Major challenges were encountered in this attempt to determine the economically optimal frequency and quantity of agricultural lime application to low pH soils used to produce continuous winter wheat. To achieve the objective it was necessary to develop a model to explain how soil pH changes over time if soils are continuously cropped but not limed. A second model was necessary to explain how soil pH

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Initial soil pH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.1</td>
</tr>
<tr>
<td>No lime applied (NPV ha(^{-1}))</td>
<td>$2852</td>
</tr>
<tr>
<td>Multiple applications (NPV ha(^{-1}))</td>
<td>$3531</td>
</tr>
<tr>
<td>First application (Mg ha(^{-1}))</td>
<td>6.38</td>
</tr>
<tr>
<td>Subsequent applications (Mg ha(^{-1}))</td>
<td>0.67</td>
</tr>
<tr>
<td>Years applied</td>
<td>1, 10, 13, 16, 19, 22, 25</td>
</tr>
</tbody>
</table>
changes over time after lime is applied. A third model was necessary to explain wheat grain yield response to soil pH.

Data were not available to estimate parameters for the model used to explain soil pH change over time in fields that are continuously cropped but not limed. For this model expert opinion was used to calibrate the parameters. While some data were available to estimate parameters for the model used to explain soil pH change in continuously cropped soils in response to liming, they had limitations. The data were obtained from a study that had been conducted on a farmer’s field. An average estimate of the soil pH across all plots was taken prior to liming. However, estimates for pH of the individual experimental plots were not taken prior to lime application. Plots of the data, as shown in Fig. 4, clearly illustrate considerable variability across replications. The study was terminated after 8 years. Based upon the data, as shown in Fig. 4, the effects of the lime applications on soil pH very likely continued beyond 8 years. However, given the lack of data, estimates of pH change resulting from lime application after the eighth year are based upon extrapolations of the data and contingent upon the form of the function.

Conducting field trials to obtain data for the models that were estimated is extremely difficult, time consuming, and expensive. On existing experiment station plots, soil pH is generally managed to ensure that soil pH is not a yield-limiting factor. Thus, if a researcher wants to conduct a soil pH study on an existing experiment station, the soil must be artificially acidified. If the soil is artificially acidified, pH response to lime may differ from that of naturally acidified soils in the region. An alternative is for the researcher to find a naturally acidified location off the station, and negotiate with the landowner for use of a uniform portion of the field. If the researcher elects to use a naturally acidified field off the station, several additional

Fig. 6. Change in soil pH over 25 years under continuous wheat production with economically optimal lime applications in years 1, 10, and 17, given an initial soil pH of 4.8.
problems may be encountered. For example, the plots may be inadvertently con-
taminated by some activity of the farmer. Land ownership may change and the new
owner may not wish to continue the study. These potential problems are of partic-
ular concern for a lime application study that under ideal situations would be
monitored for a number of years.

5. Conclusions

The underlying result of the economic model is that when the soil pH level is
below the estimated plateau level for wheat grain production, an initial application
of lime is warranted to increase the pH to reach the plateau level. Subsequent
applications are made to maintain the soil pH near that level. Lime applications
require substantial investments, especially when the initial soil pH is very low.
However, the results in Table 3 demonstrate that this strategy is economically opti-
mal over time.

Table 3 also shows that the maximum net present value of returns net of the cost
of liming decreases with the decrease in the initial soil pH. One unit of soil pH (from
4.1 to 5.2) was “worth” about $245 net present value per hectare in terms of wheat
grain production. This suggests that land appraisers should consider the soil pH
when valuing land used to produce wheat.

Although the solutions provided by the evolutionary solver seem practical, they
should be considered as “near-optimal solutions”. However, the solutions are
expected to be at least as precise as machines used to apply agricultural lime.

It is important to account for the residual effect of liming when estimating eco-
nomically optimal lime rates. In this experiment, the carryover effect of one-time
lime application lasted longer than the 8-year study. The approach used in this study
extrapolates the beneficial effects of liming over time. Unfortunately, data were not
available to test the precision of the extrapolation.

An additional shortcoming of this study is that conditions that determine soil pH
are specific for different soil types. Parameters $\alpha$ and $\beta$ in the model used to estimate
soil pH change over time [Eq. (7)] may be different for different soils under different
climatic conditions, which is a common problem for studies of carryover effects of
fertilizers.

The results showed that the optimal input rates were greatly affected by the choice
of the crop response model. The difference between the optimal rates from the
quadratic model and the plateau models was more than 2.2 Mg ha$^{-1}$ of lime. Cri-
tical pH levels derived from plateau models appear to be consistent with physi-
ological requirements of wheat.

Continuous representation of the carryover effect of lime application allows
seeing the pattern of lime application strategies. Depending on the initial soil acidity
level, the model determined an initial optimal application level and the timing
and level of follow-up applications.

Finding the optimal solution for an applied problem can be achieved either through
simplifying a system to the level of well-behaved models and using conventional
analytical optimization methods or attempting to find the optimal solution to the complex system through numerical optimization techniques. The optimization strategies based on genetic algorithms offer practical tools for non-smooth-discontinuous problems. This approach can be an appropriate alternative to dynamic programming setting, especially when a problem suffers from the curse of dimensionality.

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References